(Linear) Algebra for Program Analysis *

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I would like to report on onging research in which we use techniques from algebra and linear algebra to construct highly precise analysis routines for imperative programs or program fragments. We are interested in programs that work on variables taking values in some fixed ring or field $\mathbb{F}$, e.g., the integers or the rationals. Our analyses precisely interpret assignment statements with affine or polynomial right hand side and treat other assignment statements as well as guarded branching statements conservatively as non-deterministic statements. More specifically, we have constructed

1. an interprocedural analysis that determines for every program point of an affine program all valid affine relations[1];
2. a generalization of this analysis to polynomial relations of bounded degree[2] and to programs with local variables; and
3. an interprocedural analysis that decides validity of polynomial relations in polynomial programs.

The running time of analysis 1 and 2 is linear in the size of the program and polynomial in the number of program variables. For analysis 3 we have a termination guarantee but do not know an upper complexity bound. These analyses have many potential applications, because analysis questions can often be coded as affine or polynomial relations easily. Some obvious examples are:

- x is a constant of value $a \in \mathbb{F}$ at a program point p iff the affine relation $x - a = 0$ is valid at p;
- x and y have always the same value at p iff the affine relation $x - y = 0$ is valid at p; and
- x only takes values in the set $\{a_1, \ldots, a_k\} \subseteq \mathbb{F}$ iff the polynomial relation $(x - a_1) \cdot \ldots \cdot (x - a_k)$ is valid.

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[1] An *affine relation* is a condition of the form $a_0 + \sum_{i=1}^{n} a_i x_i = 0$, where $a_0, \ldots, a_n \in \mathbb{F}$ are constants from the underlying field and $x_1, \ldots, x_n$ are the program variables. A relation is *valid* at a program point, if it holds whenever control reaches that program point.

[2] A *polynomial relation* is a condition of the form $p(x_1, \ldots, x_n) = 0$ where $p(x_1, \ldots, x_n)$ is a multi-variate polynomial in $x_1, \ldots, x_n$ over $\mathbb{F}$. A polynomial relations is bounded by $d \in \mathbb{N}$ if the sum of exponents of every monomial in p is less than or equal d.